# A New Collection Theory of Cyclone Separators

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With the growing concern for the environmental effects of particle pollution, it becomes more and more important to be able to design optimized pollution control devices. For instance, cyclones have been used to remove dust from industrial gas streams since the late 1800's (Dirgo and Leith, 1985). Their simple design, low capital and maintenance costs, and adaptability to a wide range of operating conditions, have made cyclones the most widely used industrial dust collectors. In spite of the disadvantage of a low collection efficiency for particles smaller than about 5$\mu$m in diameter, there has been a renewed interest in cyclones, particularly in high-temperature, high-pressure applications. The recent emphasis, for example, on the development of coal-based technologies for producing electric power has created a renewed interest in cyclone separators.

Many different types of cyclones have been and will be built. The reverse-flow cyclone with a tangential inlet, Figure 1, is most often used for industrial gas cleaning. The theories of cyclone particle collection are numerous and differ greatly in complexity. Unfortunately, since the theories are not very satisfactory, the necessary design basis of cyclone separators is not readily available.

Historically, cyclones have been characterized by a cut size ($d_{50}$) which defines the particle size for which the cyclone collection efficiency is 50%. The cut size can be calculated approximately by a balance between the centrifugal force on a particle due to its angular velocity, and the drag force on the particle due to the radial gas velocity (see Stairmand, 1951; Stern, et al., 1955). Unfortunately, this procedure does not allow prediction of the shape of the grade efficiency curve. Nonetheless, this simple approach can be used to scale experimental data for geometrically similar cyclones (Stairmand, 1951; Lapple, 1951).

Different assumptions about initial radial position and residence time of particles lead to different results. The Lapple (1951) cut size theory is the most widely used example of the cut size theories. Lapple assumed that a particle entering the cyclone is evenly distributed across the inlet opening. The particle that travels from the inlet half width to the wall during the time in the cyclone is collected with 50% efficiency. Barth's (1956) collection theory is another representation of cut size theories, and is widely cited in European literature. Barth calculated the terminal settling velocity for static particles, based on the exact balance between the centrifugal force and the drag force. The collection efficiency for any particle size is determined from the ratio of its settling velocity to the terminal settling velocity of the static particle. Dirgo and Leith (1985) modified the Barth theory and found a simple expression for Barth's plot of the collection efficiency versus the ratio. A recent cyclone collection theory by Leith and Licht (1972) proposed an improved model which recognized the inherently turbulent nature of cyclones and the distribution of gas residence times within cyclones. The theory allows direct calculation of collection efficiency for particles of any size by cyclones of any design. But unfortunately, they assumed that at any height in the cyclones, uncollected dust is completely and uniformly mixed, and that the gas is uniformly mixed over a cross section, and also that a constant particle radial velocity exists within the cyclone. These assumptions regarding complete mixing are questionable, and there is experimental evidence to support the fact that there is, indeed, a concentration gradient in the radial direction of the cyclones (Hejam, 1971; Mothes and Loffler, 1982). A newer collection theory by Dietz (1981) divides the cyclone into three regions: the entrance region, the downflow (or annular) region, and the upflow (or core) region. Dietz proposed the interchange of particles between the annular and core regions. But unfortunately, Dietz also assumed that turbulence produces a uniform radial concentration profile for uncollected particles within each region. Furthermore, although Dietz proposed the reverse flow nature of the cyclone, he assumed that particle radial velocity is a constant, being the velocity for the particle at the cyclone wall. This assumption as well, should be justified.

In the present paper, a new mathematical model is developed to describe the process of particle motion in cyclones. It assumes neither a constant radial particle velocity nor uniform radial

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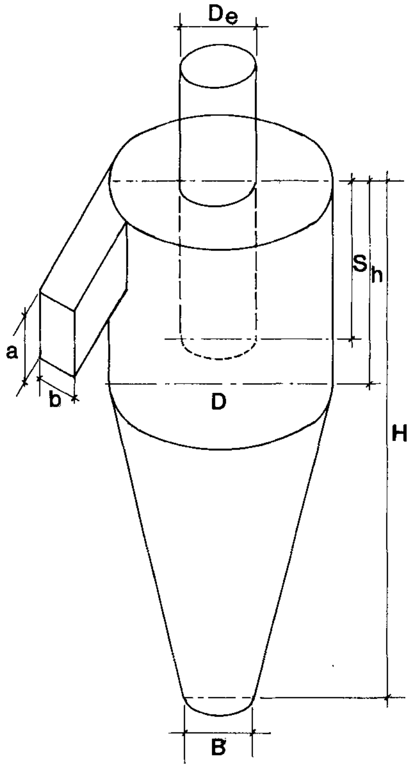


**Figure 1. Reverse-flow cyclone dimensions.**

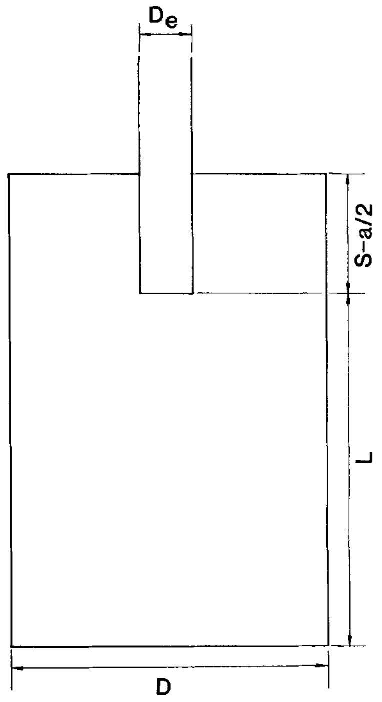


**Figure 2. Modified cyclone geometry for analysis.**

concentration profile, for uncollected particles within cyclones. The model includes particle bounce or reentrainment and turbulent diffusion at the cyclone wall. A two-dimensional analytic expression of particle distribution in the cyclone is found. Collection efficiency for particles of any size by cyclones of any design can be directly calculated from the resultant expression which shows good agreement with the modified Barth theory, and with the experimental data by Dirgo and Leith (1985).

## New Theory

To further simplify the analysis, the conventional cyclone geometry, Figure 1, will be modified to a right circular cylinder, Figure 2. The cyclone radius and exit tube radius are unchanged. The engagement length of the modified cyclone is equal to the average engagement length, ($S$-$a/2$) (Dietz, 1981).

By neglecting turbulent diffusion throughout the interior of the fluid in the cyclone, as well as the particle settling velocity in $z$ direction, and based on the equations of continuity and conservation of particles, a new model of particle distribution in the cyclone is found as

$$w\frac{\partial c}{\partial r} + u\frac{1}{r}\frac{\partial c}{\partial \theta} = 0, \quad (0 \le \theta \le \theta_1) \tag{1}$$

With the consideration of turbulent diffusion and particle bounce or reentrainment on the cyclone wall, the boundary conditions are

$$c = c_0, \quad \text{at } \theta = 0 \tag{2}$$

$$D_r\frac{\partial c}{\partial r} = (1 - \alpha)wc, \quad \text{at } r = D/2 \tag{3}$$

The turbulent diffusion coefficient was given by Taylor (1954) as

$$D_r = 0.052Ru\sqrt{f/8} \tag{4}$$

where the friction coefficient, $f$, is assumed to be 0.02 (Dirgo and Leith, 1985), and

$$R = (D - D_e)/2 \tag{5}$$

The radial particle velocity, $w$, can be directly calculated from a balance between the centrifugal force due to the gas stream, and the drag force (assuming Stokes's drag) due to the radial gas velocity. It is

$$w(r) = (\rho_p - \rho_g)u^2d^2/18\mu r \tag{6}$$

The radial dependence of the tangential velocity was given by a modified expression of the free vortex in an invisid fluid (Alexander, 1949; Caplan, 1968):

$$ur^n = u_w r_w^n = \text{constant} \tag{7}$$

Alexander (1949) presented an empirical expression to calculate the vortex exponent, $n$, for any cyclone diameter and gas temperature:

$$n = 1 - [(1 - 0.67D^{0.14})(T/283)^{0.3}] \tag{8}$$

thus

$$u(r) = \frac{(1 - n)Q}{b(r_w^{1-n} - r_n^{1-n})r^n} \tag{9}$$

The concentration distribution in a cyclone is found by solving the equations above, to give:

$$c(r,\theta) = \frac{c_0(r_w - r_n)\exp\left\{-\lambda\left[\theta - \frac{1}{K(1+n)}r^{1+n}\right]\right\}}{\int_{r_n}^{r_w}\exp\left\{\frac{1}{K(1+n)}r^{1+n}\right\}dr} \tag{10}$$

where

$$K = \frac{(1-n)(\rho_p - \rho_g)d^2Q}{18\mu b(r_w^{1-n} - r_n^{1-n})} \tag{11}$$

$$\lambda = \frac{(1-\alpha)Kw_w}{D_r r_w^n} \tag{12}$$

According to the definition of fraction efficiency of the cyclone, the cyclone collection efficiency is

$$\eta = 1 - \frac{c_1(\theta_1)}{c_0} \tag{13}$$

The average particle concentration, $c_1$ at $\theta = \theta_1$, is

$$c_1 = c_0\exp\{-\lambda\theta_1\} \tag{14}$$

Therefore, a resultant expression of the collection efficiency for particles of any size by cyclones of any design is found as

$$\eta = 1 - \exp\{-\lambda\theta_1\} \tag{15}$$

where

$$\theta_1 = 2\pi(S + L)/a \tag{16}$$

and the natural length of the cyclone, $L$, defined by Alexander (1949) as the farthest distance the spinning gas extends below the gas outlet duct, is

$$L = 2.3D_e\,(D^2/ab)^{1/3} \tag{17}$$

If the natural length > the height of the cyclone cone,

$$L = H - S \tag{18}$$

## Comparison of Theoretical and Experimental Results

A Stairmand high-efficiency cyclone separator can be characterized by eight dimensions that are often expressed as their ratio to the cyclone diameter, $D$. A recent experiment was made by Dirgo and Leith (1985) with $n = 0.56$ and $D = 0.305$ m. A Laskin nozzle aerosol generator was used to nebulize Arcoprime 200, a mineral oil with a density of 860 kg/m$^3$. The system produced spherical, liquid droplets that neither bounce nor undergo reentrainment after striking the cyclone wall. Figure 3 compares our theoretical result with the experimental data from Dirgo and Leith (1985) and several previous theories (Lapple, 1951; Barth, 1956; Leith-Licht, 1972; Dietz, 1981). (Note that the curves based on Barth's theory have been modified by Dirgo and Leith in 1985.)

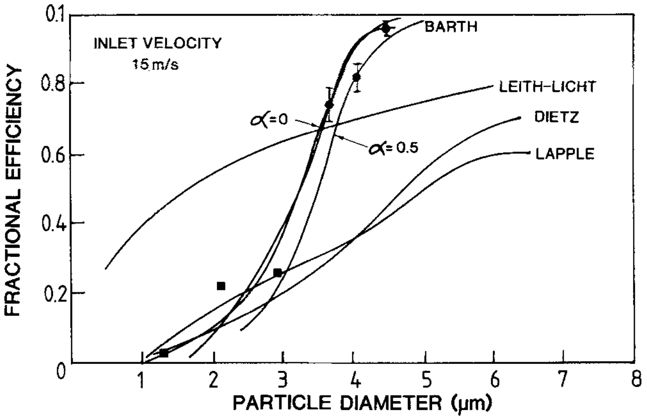


**Figure 3. Theoretical vs. experimental results.**

The comparison shows that the new theory's curves fit the data and the modified Barth theory well. If the new theory is compared with the others, it is closer than the Leith-Licht theory for larger particles, and closer than the Lapple Theory and Dietz theory for smaller particles.

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## Notation

$a$ = cyclone inlet height
$A, A_1, A_2$ = constant
$b$ = cyclone inlet width
$B$ = cyclone dust outlet diameter
$c$ = particle concentration
$c_0, c_1$ = inlet and outlet concentration of particles
$d$ = particle diameter
$D$ = cyclone body diameter
$D_e$ = cyclone gas outlet diameter
$D_r$ = radial turbulent diffusion coefficient
$d_{50}$ = cut particle diameter collected with 50% efficiency
$f$ = friction coefficient
$h$ = cyclone cylinder height
$H$ = cyclone height
$k$ = constant correlating cyclone structure and flow condition
$L$ = natural length of cyclone
$n$ = cyclone vortex exponent ($0.5 < n < 1.0$)
$Q$ = volumetric gas flow rate
$r$ = radial dimension, $r_w = D/2$ and $r_n = D_e/2$
$R$ = radius
$S$ = cyclone gas outlet duct length
$T$ = absolute temperature
$u, u_w$ = tangential component of gas velocity in cyclone vortex and at cyclone wall
$w$ = radial particle velocity
$w_n, w_w$ = radial particle velocity at $r = r_n$ and $r = r_w$
$x, y, z$ = coordinate directions

### *Greek letters*

$\alpha$ = particle bounce or reentrainment coefficient
$\lambda$ = characteristic value, Eq. 2
$\theta$ = angular coordinate
$\eta$ = efficiency
$\rho_g$ = gas density
$\rho_p$ = particle mass density
$\mu$ = gas viscosity

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